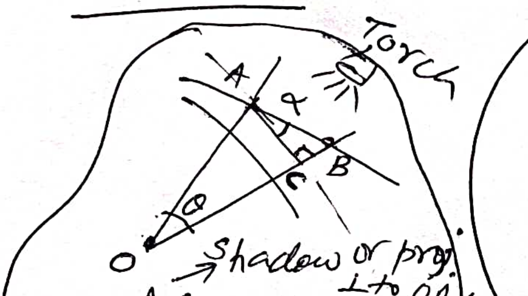
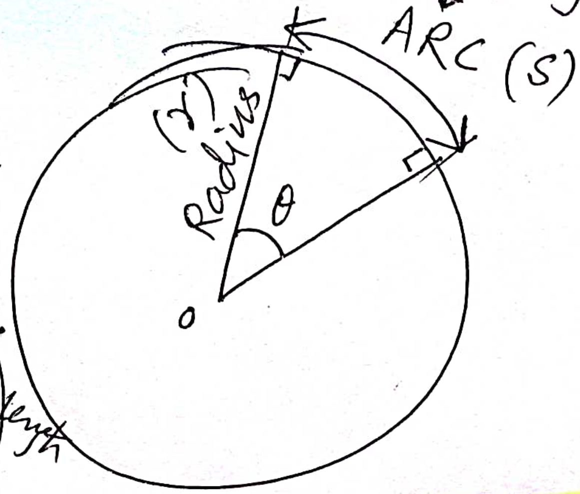
Planar $\langle$ **ANGLE** $\rangle$ Length ⊥ to Radius & along circumference

$180° = \pi$ rad

2D

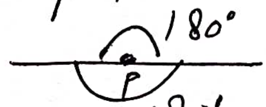


$\theta = \frac{AC}{OA} = \frac{AB \cos\alpha}{OA}$



Pi $\pi = 3.14159..$ $\simeq \frac{22}{7}$

Total angle around a point P



$= 360°$

$= 2\pi$ rad

$$\boxed{\text{angle } \theta = \frac{\text{ARC}}{\text{RADIUS}} = \frac{s}{r}}$$

Arc = Angle × radius

Σ Arc = Total Arc = Σ Angle × radius

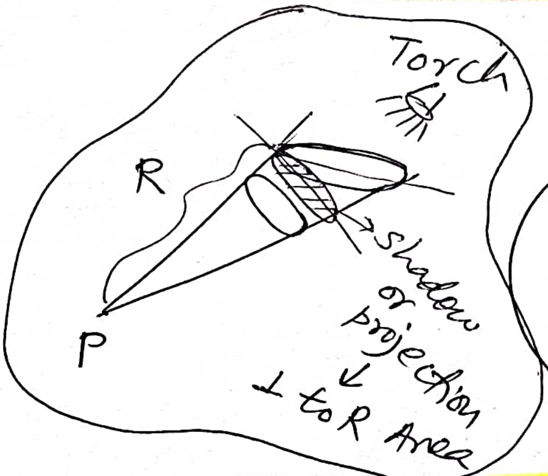
radius is constant for circle

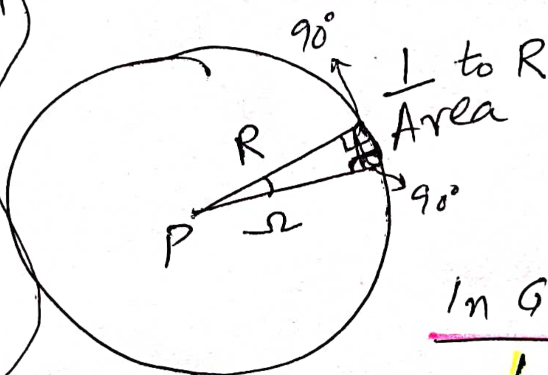
circumference of a circle OR any closed curve loop with constant distance from a fixed point $= (\Sigma \text{Angle}) \times$ radius

About a point 2π radian

$$\boxed{\text{circumference of circle} = 2\pi \times \text{radius}}$$

3D ⟨SOLID ANGLE⟩





Total solid angle around a point P $= 4\pi$ steradian

In GENERAL

$\perp$ Area $<< R^2$ to break arbitrary surface into small sphereical sections to calculate small Ω to add for final total Ω.

$$\boxed{\text{Solid angle } \Omega = \frac{\perp \text{Area}}{R^2}}$$

$$\perp \text{Area} = \text{Solid angle} \times R^2$$

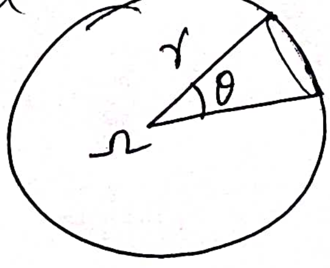
$$\sum \perp \text{Area} = \sum \text{Solid Angle} \times R^2$$

Radius is constant for sphere

$$\sum_{\text{sphere}} \perp \text{Area} = \underbrace{\left(\sum \text{Solid Angle}\right)}_{4\pi \text{ steradian}} \times R^2$$

$$\boxed{\text{Area of Sphere} = 4\pi R^2}$$

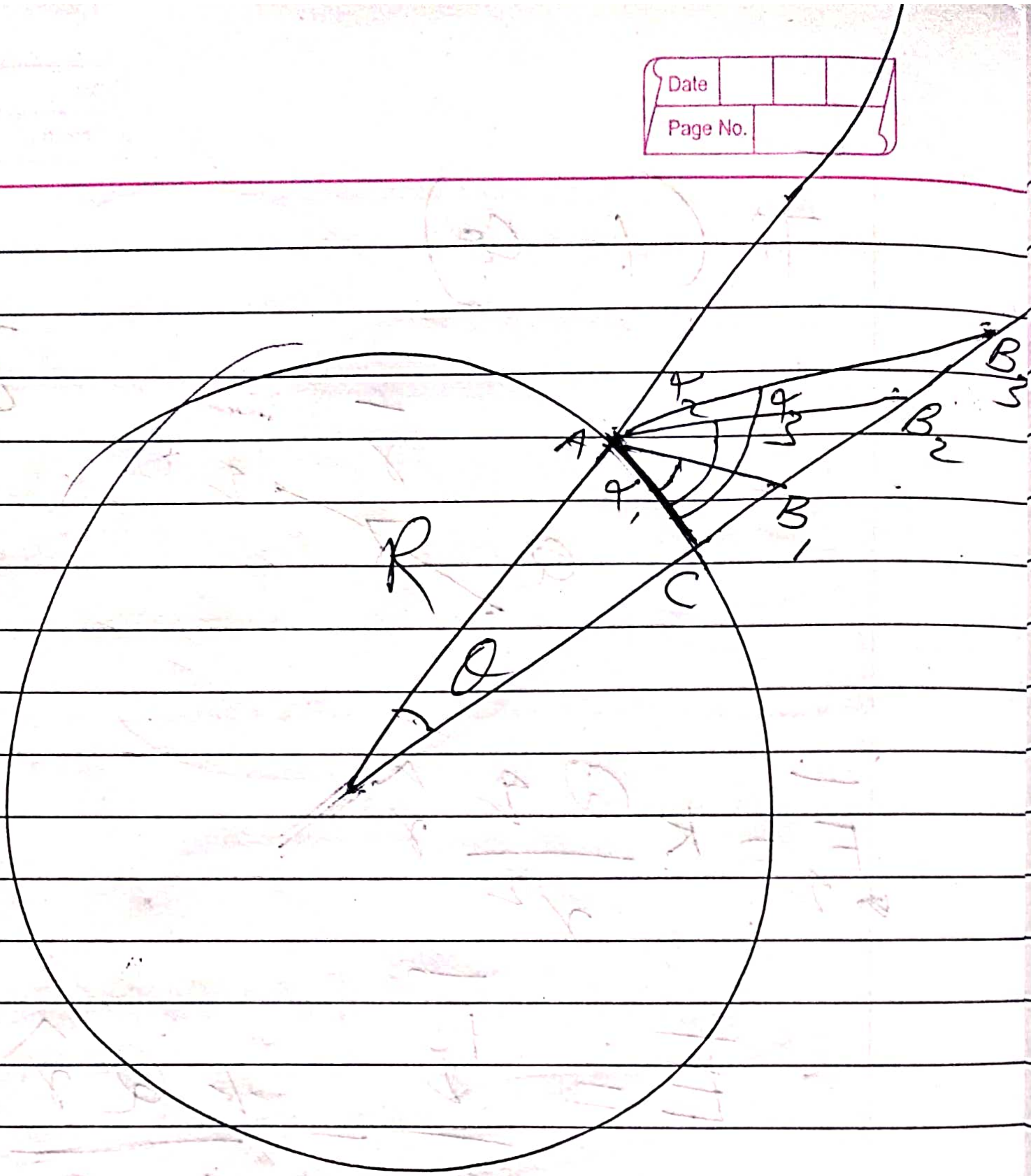
Ex Solid Angle(Ω) of a cone of angle(θ)



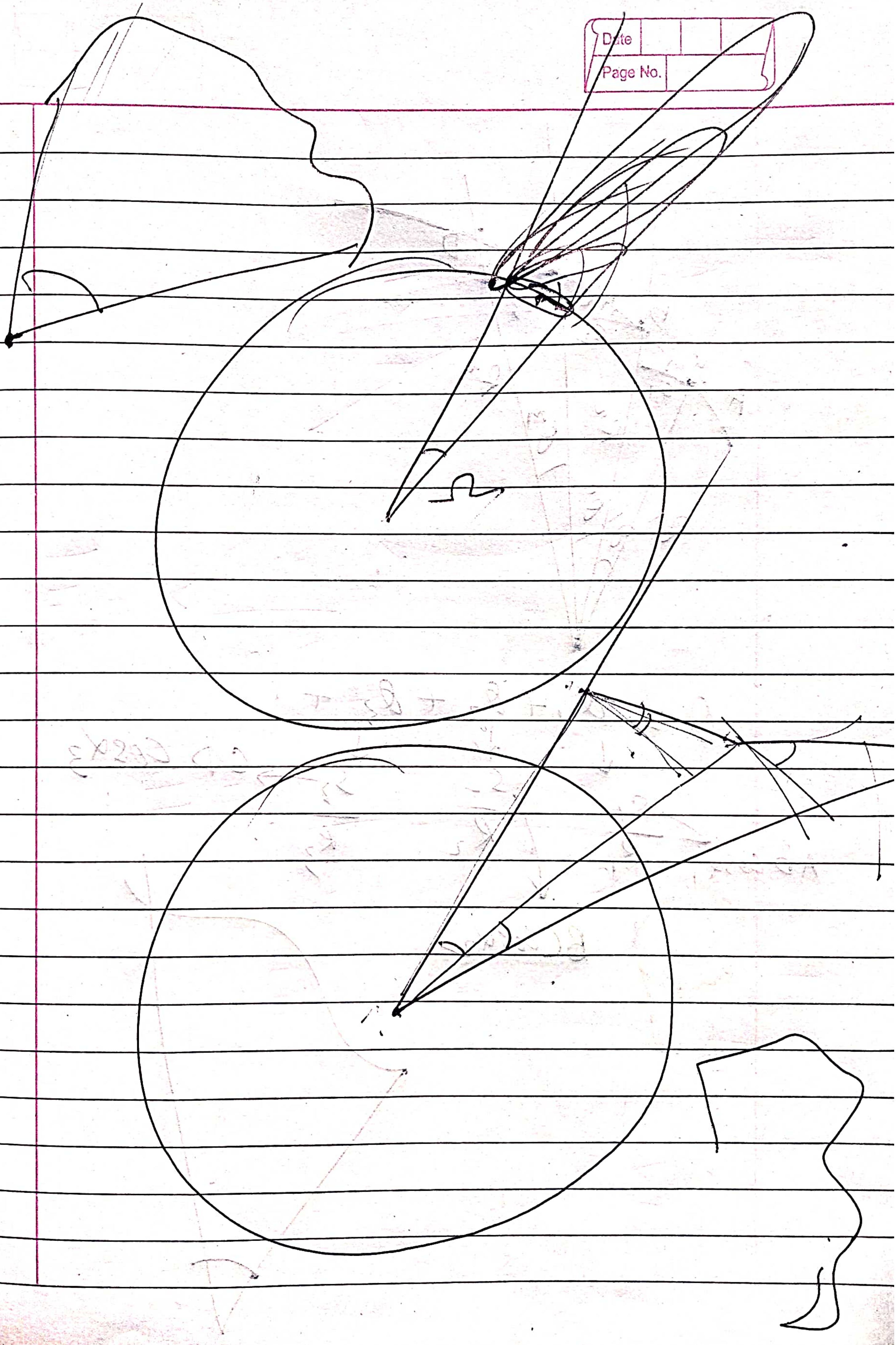
$$\Omega(\theta) = \pi\,(1 - \cos\theta)$$

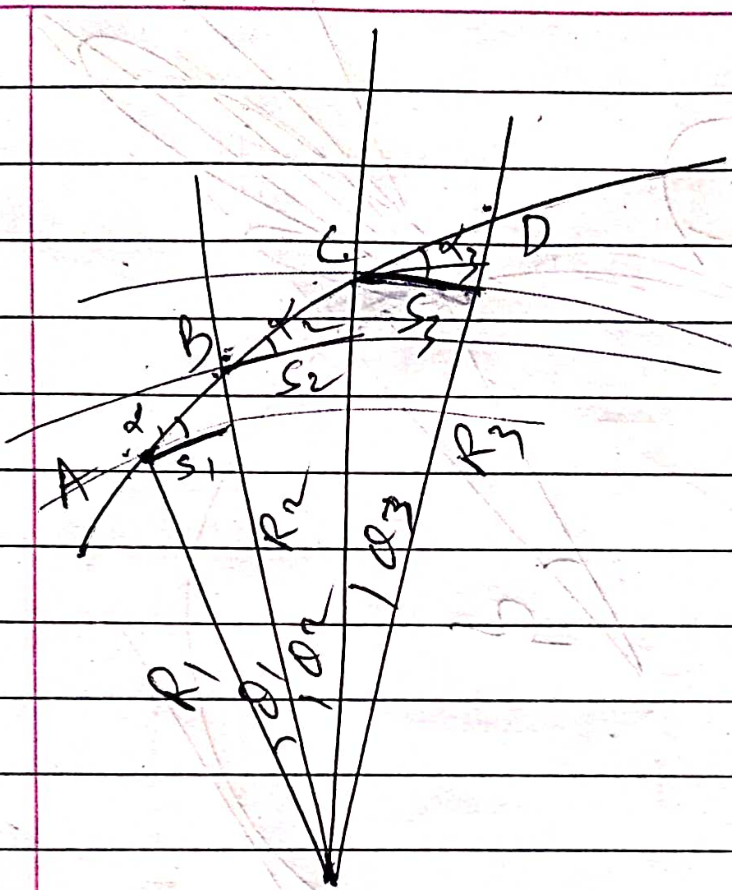
Proof by integration.

Easy verification $\Omega(0) = 0$ sterad, $\Omega\left(\frac{\pi}{2}\right) = \pi$ sterad, $\Omega(\pi) = 2\pi$ sterad



$$AB_1 \cos\alpha_1 = AC = AB_2 \cos\alpha_2$$
$$= AB_3 \cos\alpha_3$$



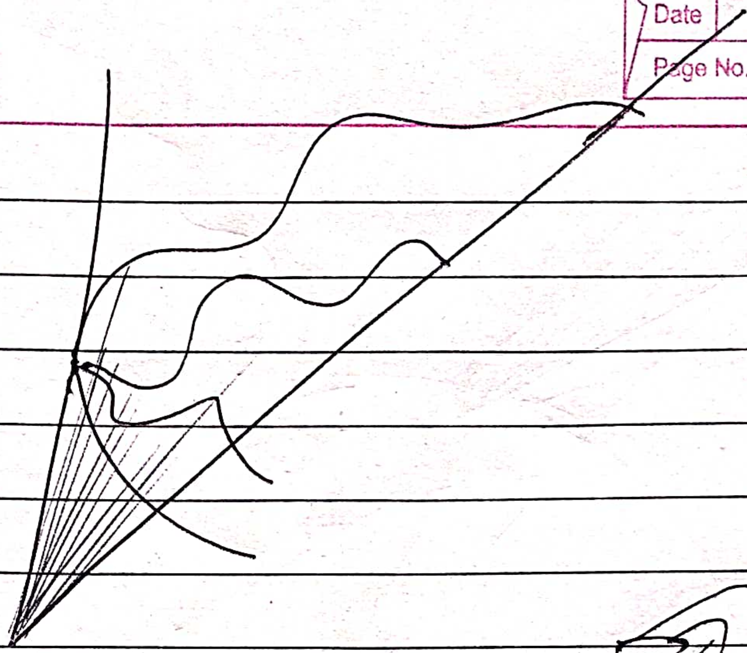


$$\theta = \theta_1 + \theta_2 + \theta_3 + \cdots$$

$\theta_1 \to \dfrac{S_1}{R_1} \to AB\cos\alpha_1$

$\theta_2 \to \dfrac{S_2}{R_2} \to BC\cos\alpha_2$

$\theta_3 \to \dfrac{S_3}{R_3}$, $S_3 \to CD\cos\alpha_3$



Flat
irregular
area