$A \quad A+D \quad A+D+D = A+2D$ <AP> $T_N = 1+(N-1)1 = N$

$\uparrow \quad \uparrow \quad \uparrow$

$1, 2, 3, 4, \ldots N, T_N$ $\quad 4 \to A+D+D+D = A+3D$

$T_1 \; T_2 \; T_3 \; T_4 \; T_5$ $\quad N^{th}$ Term $= A+(N-1)D$

$1, 3, 5, 7, 9, \ldots$ $\quad A=1, D=2$, $T_N = 1+(N-1)2$

$2, 4, 6, 8, 10, \ldots$ $\quad A=2, D=4$

$3, 6, 9, 12, 15, \ldots$ $\quad A=3, D=3$

$10, 20, 30, 40, \ldots$ $\quad A=10, D=10$

$5, 10, 15, 20, 25, \ldots$ $\quad A=5, D=5$

$\left\{ \begin{array}{l} 2, 3+2, 3+3+2, 3+3+3+2, \ldots \\ 2, 5, 8, 11, 14, 17, \ldots \end{array} \right\}$ $\quad A=2, D=3$

we already know $\boxed{1+2+3+\ldots+N = \dfrac{N(N+1)}{2}}$

sum of N natural numbers $\quad A=1, D=1, T_N = N$

<Sum of AP>

we want to find

$$S_N = \overset{T_1}{A} + \overset{T_2}{(A+D)} + \overset{T_3}{(A+2D)} + \ldots + T_N \to A+(N-1)D$$

$$= A + (A+D) + (A+2D) + \ldots + (A+(N-1)D)$$

$$= NA + \{D + 2D + \ldots + (N-1)D\}$$

$$= NA + D\underbrace{\{1+2+\ldots(N-1)\}}$$

$$\frac{(N-1)\{N-1+1\}}{2} \Rightarrow \frac{N(N-1)}{2}$$

$$\boxed{S_N = NA + \frac{DN(N-1)}{2}}$$

Sum of AP = No. of Terms × Av. of 1st and last Term

$$S_N = A + (A+D) + \cdots + \{A + (N-1)D\} = NA + \frac{DN(N-1)}{2}$$

$$S_N = \frac{2NA}{2} + \frac{DN(N-1)}{2} = \frac{N}{2}\{\underset{\substack{\downarrow \\ A+A}}{2A} + D(N-1)\}$$

$$= \frac{N}{2}\{A + \underbrace{A + (N-1)D}_{T_N \text{ Last Term}}\}$$

$$\boxed{S_N = \frac{N(A+T_N)}{2}} = N\{\text{Average of } 1^{st} \text{ and last Term}\}$$

A $\quad$ R 1×1 Ratio 1 <Geometric Progression GP>

$1, 1, 1, 1, 1, 1, \ldots$

$1, 2, 4, 8, 16, 32, 64, \ldots \quad A=1, R=2$

$1, 3, 9, 27, 81, 243, \ldots \quad A=1, R=3$

$1, 5, 25, 125, 625, \ldots \quad A=1, R=5$

$1, 10, 100, 1000, 10000, \ldots \quad A=1, R=10$

$4, 2, 1, \frac{1}{2}, \frac{1}{2}\times\frac{1}{2}, \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}, \ldots \quad A=4, R=\frac{1}{2}$

$6, 2, \frac{2}{3}, \frac{1}{3}\times\frac{2}{3}, \frac{1}{3}\times\frac{1}{3}\times\frac{2}{3} \ldots \quad A=6, R=\frac{1}{3}$

$$\underset{T_1}{A},\ \underset{T_2}{AR},\ \underset{T_3}{AR^2},\ \underset{T_4}{AR^3},\ \underset{T_5}{AR^4},\ \underset{T_6}{AR^5}, \ldots, \underset{T_N}{AR^{N-1}}$$

Sum of GP $\quad T_1 + T_2 + \cdots + T_N$

$$S = A + AR + AR^2 + AR^3 + \cdots + AR^{N-1}$$

$$-RS = -(AR + AR^2 + AR^3 + AR^4 + \cdots + AR^{N-1} + AR^N)$$

$$S - RS = A - AR^N$$

$$S(1-R) = A(1-R^N) \Rightarrow \boxed{S = \frac{A(1-R^N)}{1-R}}$$

Ex

$A=1$
$R=\frac{1}{2}$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots + \infty$ terms

$N = \infty$

$S = 2 \Leftarrow 2S = 2 + 1 + \frac{1}{2} + \cdots = 2 + S$