< Electric Force, Field, Lines of force and Flux >

Electric Force between charges Q, q at r distance:

- $\sim Q$
- $\sim q$
- $\sim \frac{1}{r^2}$

$\rightarrow F_E \sim \frac{Qq}{r^2}$

by experiment we get constant of proportionality K_E

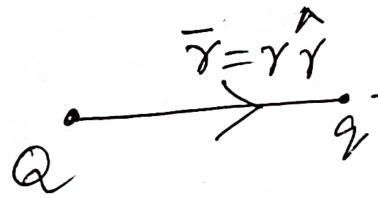
$$F_E = K_E \frac{Qq}{r^2}$$

$K_E = \frac{1}{4\pi\epsilon_0}$

F_E is Along the line joining the charges

dir^n:
- attractive For opp. charge types $+/-$
- Repulsive for same charge type $+/+$, $-/-$

combining magnitude and dir^n we get VECTOR

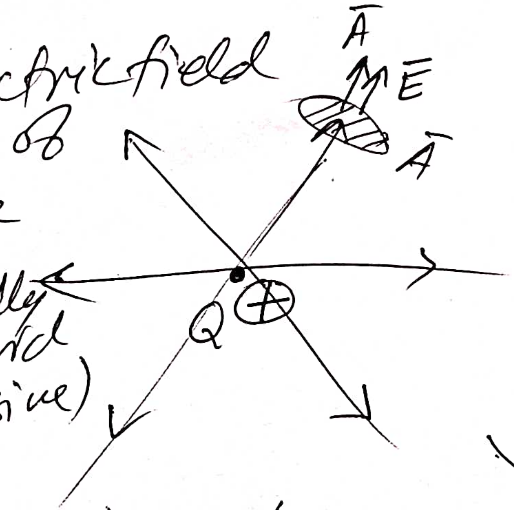


$$\rightarrow \bar{F}_E = F_E\,\hat{r} = K_E Qq \frac{\hat{r}}{r^2}$$

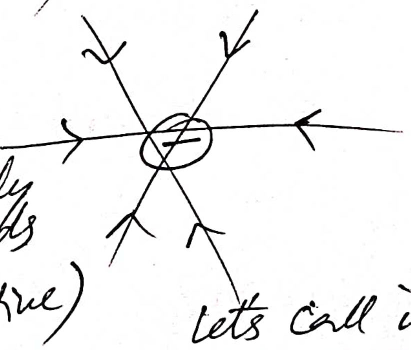
By unitary method we get $\bar{F}_E$ per unit charge ($q = 1$) called Electric field $\bar{E}$

$$\bar{E} = \frac{\bar{F}_E}{q} = K_E Q \frac{\hat{r}}{r^2}$$

Electric field Lines of Force



radially outward (Repulsive)



Radially inwards (Attractive)

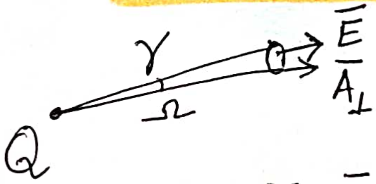
→ spherical symmetry because all dir^n are same

Dir^n of $\bar{E}$ and area vector on each point of sphere is same ⊕ or opposite ⊖

So why not try? ←

lets call it Electric FLUX $\phi_E = \bar{E} \cdot \bar{A}$ and see what happens!

< Gauss's Law > from Coulomb's Law



Flux for $\bar{A}_\perp$: $\phi_E = \bar{E} \cdot \bar{A}_\perp = E A_\perp \cos\theta$ ($\cos\theta \to 1$) $= K_E Q \Omega \to \phi_E \propto Q$, $\propto \Omega$

$A_\perp \to \Omega r^2$

$E \to \dfrac{K_E Q}{r^2}$, $K_E \to \dfrac{1}{4\pi\epsilon_0}$

Total flux around Q:

$\phi_E(\text{total}) = K_E Q_{enclosed}\, \Omega_{total}$

$\Omega_{total} \to 4\pi$ steradian

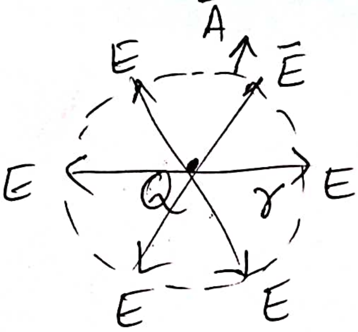
Surface closed → Total ϕ_E → Enclosed Q

Gauss's Law / Theorem

$$\phi_E(\text{total}) = 4\pi K_E Q_{enclosed} = \frac{Q_{enclosed}}{\epsilon_0}$$

$K_E \to 1/4\pi\epsilon_0$

< Coulomb's Law > from Gauss's Law



E is spherically symmetric → Gaussian surface is a sphere with charge at centre of sphere

Gauss's law

$$\phi_{Total} = \frac{Q_{enclosed}}{\epsilon_0}$$

$Q_{enclosed} \to Q$

$\phi_{Total} \to \sum \bar{E} \cdot \bar{A}$

$E A \cos\theta$ ($\cos\theta \to 1$)

$\Rightarrow \sum EA \Rightarrow E \sum A \Rightarrow E\, 4\pi r^2$

E is same on sphere

$$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$ Coulomb's Law

Gauss's Law + Symmetry ⇒ E

Symmetry → decides Gaussian Surface