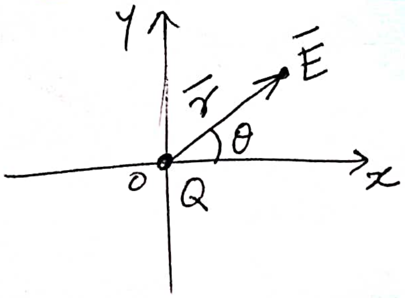
⟨physical (finite d & Q) Electric Dipole⟩



$$\bar{E} = \frac{KQ\,\bar{r}}{r^3} = \frac{KQ\,\hat{r}}{r^2}$$

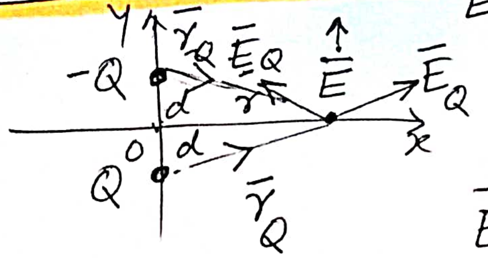
unit vector

$$\hat{r} = \begin{pmatrix} c\theta \\ s\theta \end{pmatrix} = \hat{1}(\theta)$$

Physical Dipole (Both d and Q are finite)

⟨E on EQUITORIAL PLANE⟩

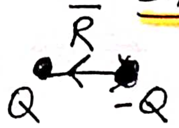
$\bar{E}$ on ~~Axis~~ EQUITORIAL plane of Dipole



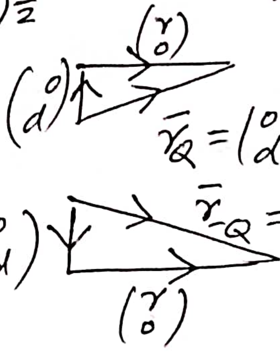
$$\bar{E} = \bar{E}_Q + \bar{E}_{-Q} = \frac{KQ\,\bar{r}_Q}{r_Q^3} + \frac{K(-Q)\,\bar{r}_{-Q}}{r_{-Q}^3}$$

$$\bar{E} = \frac{KQ}{(r^2+d^2)^{\frac{3}{2}}}\left(\bar{r}_Q - \bar{r}_{-Q}\right) = \frac{KQ}{(r^2+d^2)^{\frac{3}{2}}}\begin{pmatrix} 0 \\ 2d \end{pmatrix}$$

Dipole moment



$$\bar{p} = Q\bar{R}$$



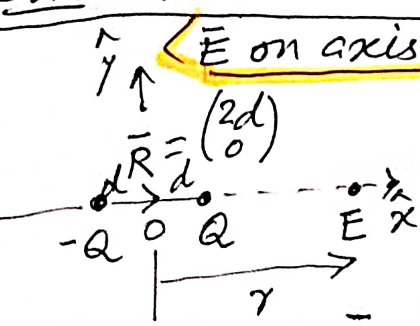
$$\bar{r}_Q = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} r \\ 0 \end{pmatrix} = \begin{pmatrix} r \\ d \end{pmatrix}$$

$$\bar{r}_{-Q} = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} r \\ 0 \end{pmatrix} = \begin{pmatrix} r \\ -d \end{pmatrix}$$

$$\boxed{\bar{E} = \frac{-K\bar{p}}{(r^2+d^2)^{3/2}}}$$

Case Iff $r \gg d$ $\quad \bar{E} = \dfrac{-K\bar{p}}{r^3}$

⟨E on axis of Dipole⟩



$$\bar{E} = \bar{E}_{-Q} + \bar{E}_Q = \frac{K(-Q)\,\bar{r}_{-Q}}{r_{-Q}^3} + \frac{KQ\,\bar{r}_Q}{r_Q^3}$$

$\bar{r}_{-Q} = \begin{pmatrix} r+d \\ 0 \end{pmatrix}$ $\qquad$ $\bar{r}_Q = \begin{pmatrix} r-d \\ 0 \end{pmatrix}$

$$= KQ\left(-\frac{\begin{pmatrix} r+d \\ 0 \end{pmatrix}}{(r+d)^3} + \frac{\begin{pmatrix} r-d \\ 0 \end{pmatrix}}{(r-d)^3}\right)$$

$\bar{p} = 2Qd\,\hat{x}$

$\bar{E} = E\,\hat{x}$

$$\frac{E}{KQ} = -\frac{1}{(r+d)^2} + \frac{1}{(r-d)^2} = \frac{(r+d)^2 - (r-d)^2}{(r+d)^2(r-d)^2} = \frac{4rd}{(r^2-d^2)^2}$$

$$E = \frac{KQ\,4rd}{(r^2-d^2)^2} = \frac{K(2Qd)\,2r}{(r^2-d^2)^2} = \frac{K2pr}{(r^2-d^2)^2}\ ; \quad \bar{E} = E\,\hat{x} = \frac{2\bar{p}rK}{(r^2-d^2)^2}$$

Case $r \gg d$ $\qquad E = \dfrac{2pK}{r^3}$

<INVERSE SQUARE LAW $F \propto 1/r^2$> Field

(shown for Q, Valid for M also) <Gauss's Law $\Rightarrow$ E>

$$\phi_{E\,\text{Total}} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

STEPS

Charge distribution → symmetry → Gaussian Surface to enclose charge

Point charge | spherical

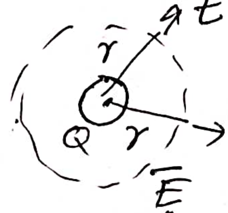
Q •

$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

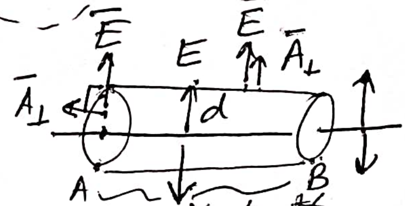
$$\boxed{E(r) = \frac{Q}{4\pi\epsilon_0 r^2}}$$

spherical charge | spherical

Q ○



Line charge | cylindrical



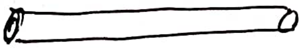
λ = Linear charge density
= charge per unit length

$$\phi_{E\,(Tot)} = \phi(\text{Flat surf.}) + \phi_{\text{curved surf.}}$$

Flat surf.: $\bar{E} \perp \bar{A}_\perp \rightarrow 0$

Curved surf.: $E(2\pi d \times 1) = \frac{\lambda}{\epsilon_0}$

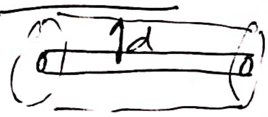
cylindrical charge | cylindrical



λ linear charge density

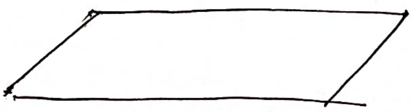
E(d)

$$\boxed{E = \frac{\lambda}{2\pi d \epsilon_0}}$$

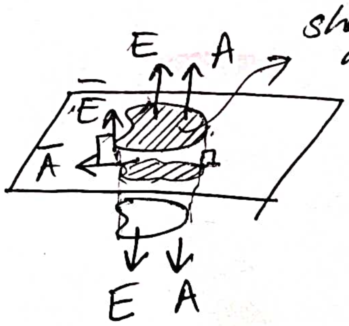


surface charge | planar

σ = Areal charge density



σ = charge per unit area



shaded area is unit area say

$$\phi_{E}(\text{Tot}) = \phi(\text{flat surf}) + \phi(\text{curved surf})$$

$$\frac{\sigma A}{\epsilon_0} \qquad EA + EA \qquad \bar{E} \perp \bar{A} \rightarrow 0$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Independent of distance from plane

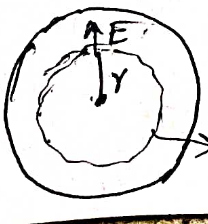
Volume charge density | sphere

ρ = charge per unit volume

$$\phi_E(\text{Tot}) = 4\pi r^2 E = \frac{\left(\frac{4\pi}{3} r^3 \rho\right)}{\epsilon_0}$$

$$\boxed{E(r) = \frac{\rho r}{3\epsilon_0}}$$

spherical symmetry



<Inverse Square Laws>

<<Gravitation (always Attractive) and Coulomb Force are both $\frac{1}{r^2}$>>

COE → Conservation of Energy

Force per unit Qty = Field	~ Quantity
Energy per unit Qty = Potential	~ Along $\bar{r}$

Prop. const.

$K_E = 1/4\pi\epsilon_0$

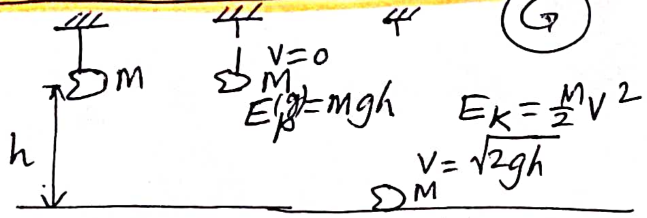
$K_G = G$

Gauss's Law: $\boxed{|\Phi(\text{Total})| = 4\pi K \text{ Quantity (enclosed)}}$

<<ENERGY>>

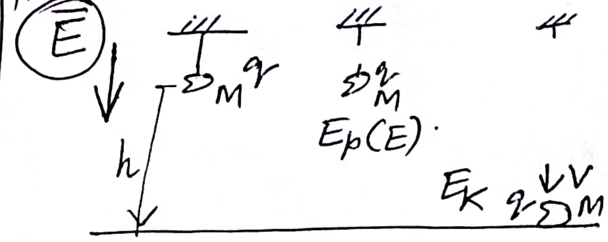
<Uniform Fields>

Ⓖ field $g\downarrow$



COE → Energy $E_K = E_p^{(g)} \Rightarrow \frac{M}{2}v^2 = Mgh$

Ⓔ field



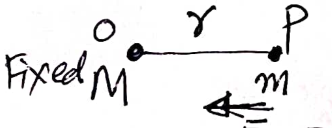
COE → Energy $E_K = E_p(E)$

$E_K \downarrow \frac{M}{2}v^2$, $E_p(E) \downarrow qEh$

<<ENERGY>>

<Non uniform fields>

Ⓖ



$F_G = \frac{GMm}{r^2}$

⊖ Sign for attractive

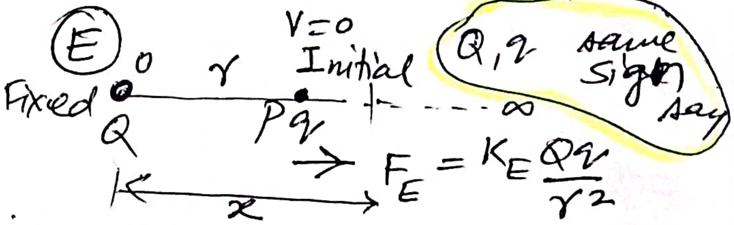
$Q \to M$

$\boxed{V_G = -\frac{K_G M}{r}}$ → scalar

Gravitational Potential due to M at Point P at distance r

~~calculus~~

Ⓔ



$F_E = K_E \frac{Qq}{r^2}$

CoE → $E_{\text{Initial}} = E_\infty$

$E_K(\text{init}) + E_p(\text{init}) = E_K(\infty) + E_p(\infty)$

$E_K(\text{init}) \to 0$; $E_p(\infty)$ → no force → 0

$$\int_{x=r}^{x=\infty} F_E(x)\,dx = K_E Qq \int_r^\infty \frac{dx}{x^2}$$

$$-\frac{1}{x}\Big|_r^\infty \to \frac{1}{r}$$

$E_p(\text{init}) = E_K(\infty) = \frac{K_E Qq}{r}$

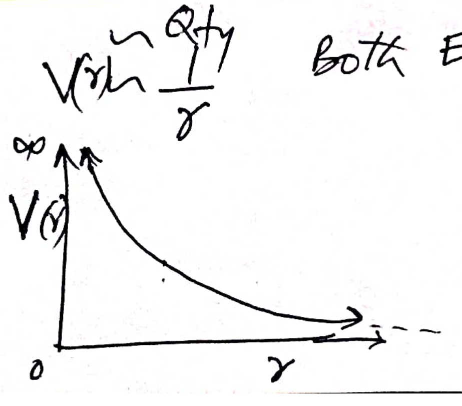
Potential

$$\boxed{V_E = \frac{E_p(\text{init})}{q} = \frac{K_E Q}{r}}$$

→ scalar

Electric Potential due to Q at distance r.

$V(r) \sim \frac{1}{r}$ (Qty) Both E and G



$\langle$ Math Study of change $\rangle$ CALCULUS

Differential

<u>case ①</u>

Finite change

$F(\Delta + x) = F(x) + \Delta F \rightarrow \boxed{\underset{\text{change}}{\Delta F} = \underset{\text{Final}}{F(x+\Delta)} - \underset{\text{Initial}}{F(x)}}$

<u>case ②</u> $\Delta \rightarrow 0$

very small or Infinitesimal change $\Delta \rightarrow dx$

DIFFERENTIAL of x

$$\boxed{dF(x) = F(x+dx) - F(x)}$$

$F(x) = A \rightarrow dF(x) = \boxed{dA = 0}$ Because A is Constant

$F(x) = x \rightarrow d\underbrace{F(x)}_{x} = \underbrace{F(x+dx)}_{x+dx} - \underbrace{F(x)}_{x} = dx$

$F(x) = x^2 \rightarrow dx^2 = \underbrace{F(x+dx)}_{(x+dx)^2} - x^2 = \{\cancel{x^2} + 2x\,dx + \overset{0}{(dx)^2}\} - \cancel{x^2}$

$$\boxed{d.(x^2) = 2x\,dx}$$

$F(x) = \frac{1}{x} \rightarrow d\left(\frac{1}{x}\right) = \underbrace{F(x+dx)}_{\frac{1}{x+dx}} - \frac{1}{x} = \frac{\cancel{x} - \cancel{x} - dx}{\underbrace{(x+dx)}_{x \text{ as } dx \to 0}x} = \frac{-dx}{x^2}$

$F(x) = \frac{1}{x^2} \rightarrow d\left(\frac{1}{x^2}\right) = \underbrace{F(x+dx)}_{\frac{1}{(x+dx)^2}} - \frac{1}{x^2} = \frac{x^2 - (x+dx)^2}{x^2\underbrace{(x+dx)^2}_{x^2 \text{ as } dx \to 0}} = \frac{-2x\,dx}{x^4} = \frac{-2dx}{x^3}$

$F(x) = x^3 \rightarrow d(x^3) = \underbrace{F(x+dx)}_{(x+dx)^3} - x^3 = \cancel{x^3} + 3x^2dx + \underbrace{3x(dx)^2 + (dx)^3}_{\to 0 \text{ as } dx \to 0} - \cancel{x^3}$

$= 3x^2 dx$