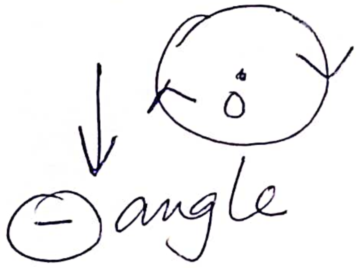
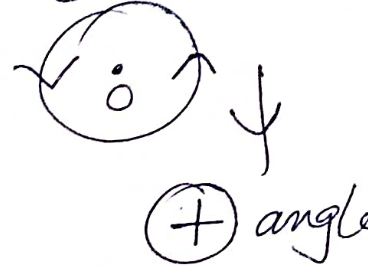
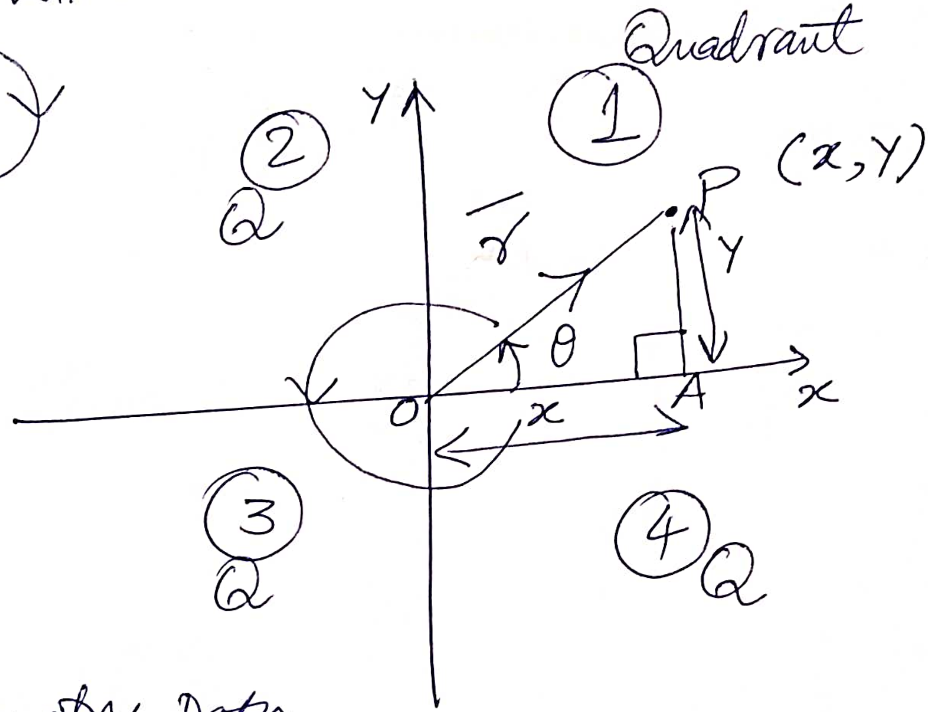
⟨Geometry Defⁿ⟩

Clockwise



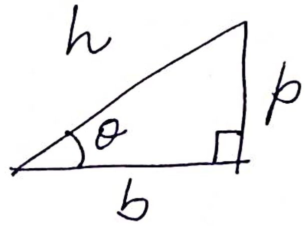
Anti clockwise





Trigonometry Defⁿ

Sin	Cos	Tan
$\frac{p}{h}$	$\frac{b}{h}$	$\frac{p}{b}$



Cartesian coordinates

$\frac{x}{OP} = \cos\theta \rightarrow x = OP\cos\theta$; $\frac{Y}{OP} = \sin\theta \rightarrow Y = OP\sin\theta$

Vector $\overset{\text{Def}^n}{=}$ Magnitude AND Direction

$\bar{r} = |\bar{r}| \times \hat{r} = r\hat{r}$

$\bar{r}$: Cartesian $\begin{pmatrix} x \\ y \end{pmatrix}$; Polar (r, θ)

$|\bar{r}| = r$

Unit vector $\hat{r} = \frac{\bar{r}}{r} = \frac{\begin{pmatrix} x \\ Y \end{pmatrix}}{r} = \begin{pmatrix} x/r \\ Y/r \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Ex $x = 3, Y = 4 \Rightarrow \hat{r} = \begin{pmatrix} x/r \\ Y/r \end{pmatrix} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} = \begin{pmatrix} .6 \\ .8 \end{pmatrix}$

unit vector in θ direction

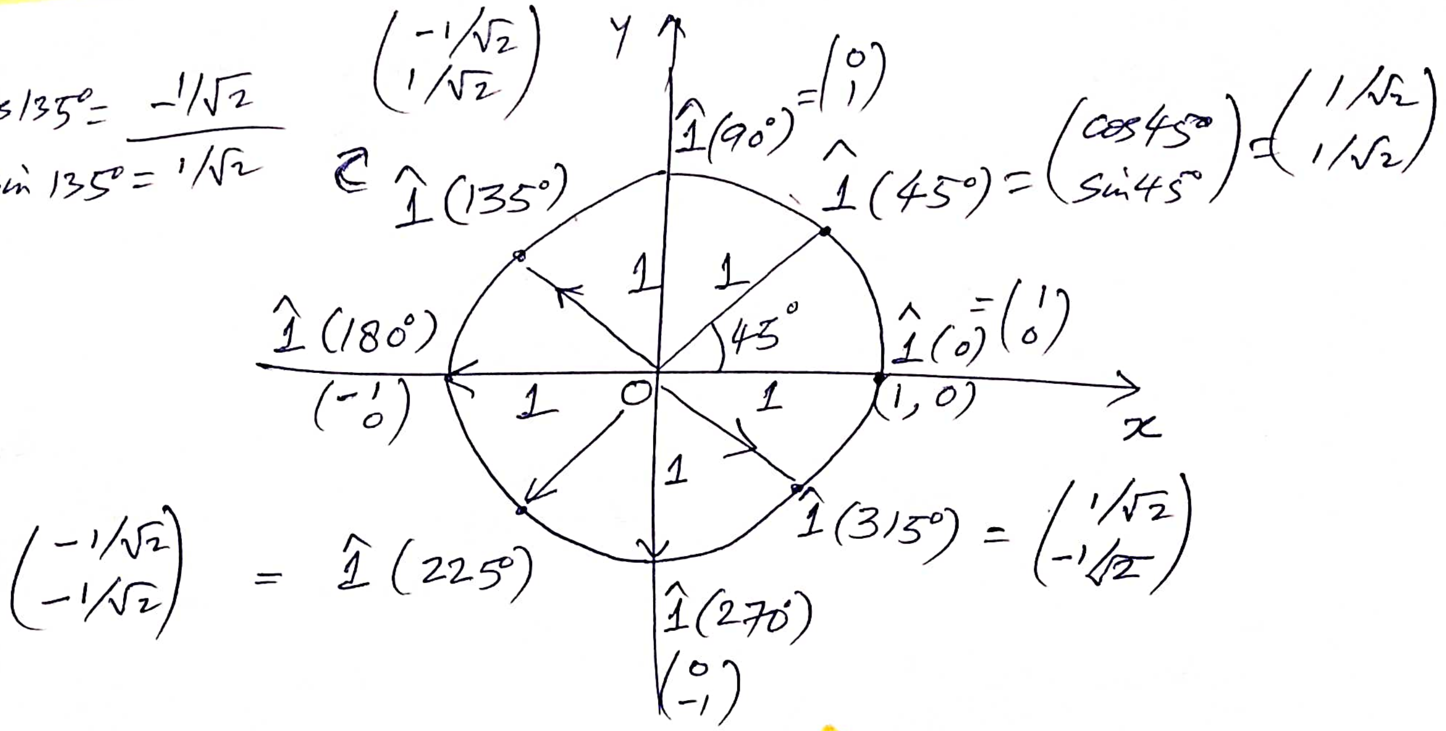
$$\hat{1}(\theta) = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

< unit circle >

unit vectors starting at 0° with 45° difference

$\cos 135° = -1/\sqrt{2}$

$\sin 135° = 1/\sqrt{2}$



< Basic Sin / cos / Tan Values > Easy way

$$\sqrt{\frac{0}{4}} \quad \sqrt{\frac{1}{4}} \quad \sqrt{\frac{2}{4}} \quad \sqrt{\frac{3}{4}} \quad \sqrt{\frac{4}{4}}$$

$$0 \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{2} \quad 1$$

sin S →	0°	30°	45°	60°	90°
← C cos	90°	60°	45°	30°	0°

Ex

$$\tan = \frac{\sin}{\cos} = \frac{S}{C} \;; \quad \tan 0° = \frac{\sin 0°}{\cos 0°} = \frac{0}{1} = 0$$

$$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \;; \quad \tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{1/\sqrt{2}}{1/\sqrt{2}} = 1$$

<2D - Dot product of vectors>

$$\bar{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = a_x \hat{x} + a_y \hat{y}$$

$$\bar{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix} = b_x \hat{x} + b_y \hat{y}$$

$$\bar{a} \cdot \bar{b} = (a_x \hat{x} + a_y \hat{y}) \cdot (b_x \hat{x} + b_y \hat{y})$$

$$= a_x b_x (\hat{x} \cdot \hat{x}) + a_x b_y (\hat{x} \cdot \hat{y}) + a_y b_x (\hat{y} \cdot \hat{x}) + a_y b_y (\hat{y} \cdot \hat{y})$$

($\hat{x} \cdot \hat{x} \to 1$, $\hat{x} \cdot \hat{y} \to 0$, $\hat{y} \cdot \hat{x} \to 0$, $\hat{y} \cdot \hat{y} \to 1$)

$$\boxed{\bar{a} \cdot \bar{b} = a_x b_x + a_y b_y}$$

$$\bar{a} \cdot \bar{b} = a\, b \cos\theta$$

$a \to \sqrt{a_x^2 + a_y^2}$, $b \to \sqrt{b_x^2 + b_y^2}$

Ex Find angle between $\bar{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$; $\bar{b} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$?

Sol^n^

$$\bar{a} \cdot \bar{b} = a_x b_x + a_y b_y = 3 \times (-4) + 4 \times 3 = -12 + 12 = 0$$

$$= a\, b \cos\theta$$

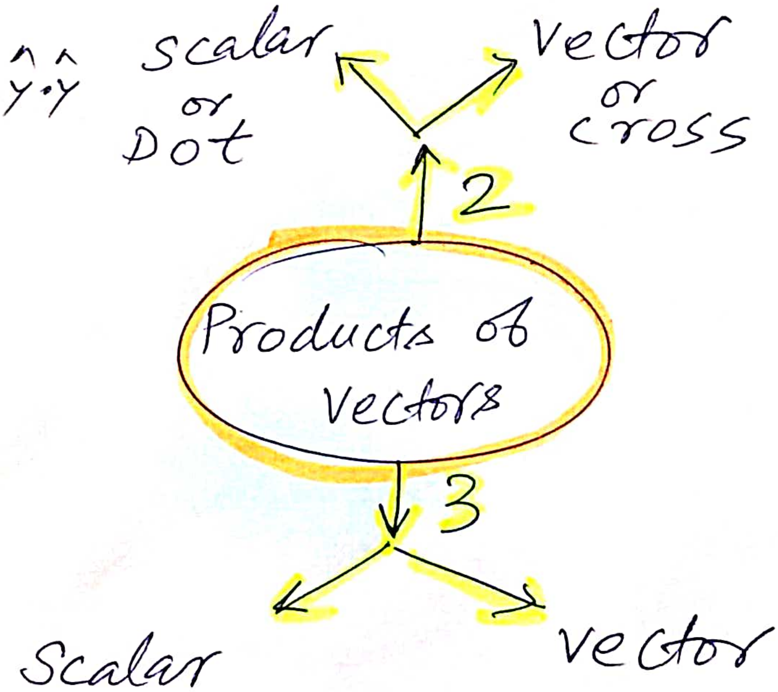
$$\Rightarrow \cos\theta = \frac{0}{ab} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

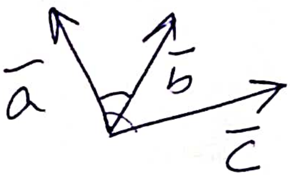
$$\bar{a}\cdot\bar{b} = a\,b\cos\theta$$

$$\hat{x}\cdot\hat{x} = 1 = \hat{y}\cdot\hat{y}$$
$$\hat{x}\cdot\hat{y} = 0$$

$$\bar{a}\times\bar{b} = a\,b\sin\theta\,\hat{1}$$

$$\hat{x}\times\hat{x} = 0 = \hat{y}\times\hat{y}$$
$$\hat{x}\times\hat{y} = \hat{z}$$





$$(\bar{a}\times\bar{b})\cdot\bar{c}$$

volume

$$\bar{a}\times(\bar{b}\times\bar{c})$$
$$= \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b})$$